Perspective: Photonic flatbands

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Perspective: Photonic flatbands

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Flatbands are receiving increasing theoretical and experimental attention in the field of photonics, in particular in the field of photonic lattices. Flatband photonic lattices consist of arrays of coupled waveguides or resonators where the peculiar lattice geometry results in at least one completely flat or dispersionless band in its photonic band structure. Although bearing a strong resemblance to structural slow light, this independent research direction is instead inspired by analogies with "frustrated" condensed matter systems. In this Perspective, we critically analyze the research carried out to date, discuss how this exotic physics may lead to novel photonic device applications, and chart promising future directions in theory and experiment.
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I. INTRODUCTION

Analogies between electronic condensed matter systems and optics attract widespread attention nowadays, underlying much research in photonics. Prominent examples where such analogies have been fruitfully developed include photonic crystals,[1] random lasers,[2] and photonic topological insulators.[3] In this Perspective, we would like to introduce and flatter a new member of this growing family: photonic flatbands.

Flatband lattices—periodic media with at least one completely dispersionless Bloch band—have been known since the 1980s,[4–7] but at the time many dismissed them as either pathological (requiring an intricate lattice design and being unstable against perturbations) or trivial (within an ideal flatband, all states are degenerate, so there are no dynamics). Indeed, the most obvious way to create a flatband is to take the Hamiltonian of any periodic system, compute its band structure, and divide it by the wavevector-dependent energy of one of its bands. While the newly obtained Hamiltonian will have a perfectly flat energy band, when Fourier transformed back to real space, it will typically have a complicated structure including fine-tuned long range couplings and be dismissed by anyone looking for proper candidate models to fabricate.

More recently, however, interest in flatbands has been rekindled in various areas of physics, including cold atoms, electronic condensed matter systems, and photonics, since the community became aware of a growing number of examples of simpler flatband models with short range connectivities; advanced fabrication techniques allow these "pathological" models to finally be realized in the laboratory. Moreover, they can display fascinating behavior when perturbations such as interactions are introduced. The last five years have seen several workshops and meetings dedicated to flatband phenomena,[8–11] and this trend is bound to continue in concert with experimental advances. We recently published a more general review article briefly recounting the exploration of artificial flatbands in various areas of physics since the 1980s.[12] This focus of the present article is to provide a more comprehensive discussion of details specific to photonics and our opinion on where this field is headed.

Within photonics, studies of flatbands have largely been focused on exploring their fundamental properties. For this research area to blossom into something of wider interest, a pressing issue is to explore potential applications of flatbands such as slow light or novel photonic crystal fibres, which are outside our direct expertise. Therefore, our aim here is to summarise the theoretical and experimental progress to date and discuss some open research directions to encourage other researchers to attack in these directions.

We begin in Sec. II with a brief review of the theory behind flatband lattices and their interesting properties. Next, in Sec. III, we evaluate the three settings in photonics where the most experimental progress has been made to date—waveguide arrays, exciton-polaritons in structured microcavities, and metamaterials—and suggest future directions which we believe are most promising to pursue with each. In Sec. IV, we advertise experimental platforms which are now ripe for implementing flatbands and also hold promise for device applications: coupled resonator lattices, photonic crystals, and superconducting microwave circuits. Section V concludes the article.

II. THEORY

Wave propagation in periodic media can be characterised by the Bloch band structures $\omega_m(\boldsymbol{k})$, where the eigenmode frequency (energy) ω depends on a continuous crystal momentum $\boldsymbol{k}$ and a discrete band index m. By tuning the properties of the periodic medium, one can control the dispersion relation $\omega_m(\boldsymbol{k})$, related quantities such as the wave group velocity, $\boldsymbol{v}_G \equiv \nabla_{\boldsymbol{k}} \omega_m$, and induce geometric and topological phases for light.[3,13] One of the most striking effects resulting from periodicity is the presence of local extrema in the dispersion relation, where the group velocity vanishes, $\boldsymbol{v}_G \to 0$. Such extrema are associated with slow light and wave localization.

There are two conventional approaches for achieving the vanishing group velocity using periodic photonic media. The first is based on photonic bandgaps occurring in the vicinity of the Bragg resonances of periodic structures forming *photonic crystals*.[1] In a bandgap, no propagating waves are supported, which allows defects within the structure to host localized modes and trap light, the basis for photonic crystal waveguides. In the vicinity of the band edges, the propagating waves' group velocity approaches zero, resulting in structurally induced slow light (in contrast to intrinsic slow light, e.g., electromagnetically induced transparency).[14,15] As a resonant scattering process between the forward- and backward-traveling Bloch waves, these effects are limited to a narrow range of wavevectors close to a reciprocal lattice vector. With this approach, the degree of localization or group velocity reduction can be improved by increasing the dielectric contrast of the structure, which couples forward and backward waves more strongly.

The second class of conventional structural slow light systems is based on coupled waveguides or resonators. Each individual element or "site" hosts localized modes, forming a *photonic lattice*[16,17] with a period (at least for dielectric structures) much larger than the operating wavelength, on the order of tens to hundreds of micrometers for optical wavelength devices (c.f. photonic crystals with periodicity on the order of hundreds of nanometers). Such photonic lattice-based approaches can reduce the group velocity over the entire Brillouin zone because they are not limited to the Bragg resonance; the group velocity reduction is enhanced by coupling the individual elements more weakly. On the other hand, there is a clear trade-off between this group velocity reduction and the operating bandwidth; the operating bandwidth is proportional to the coupling strength. In fact, under fairly general assumptions, quantitative bounds to slow light in both classes of systems can be derived, assuming one-dimensional propagation.[18]

Flatband lattice-based wave localization or slow light can be considered a hybrid of the above approaches that is possible in 2D or quasi-1D systems. In a flatband lattice, the vanishing group velocity is induced by interference between two or more channels or propagation paths, with the Bragg resonance condition effectively satisfied transverse to the propagation direction. This results in a distinctly different behavior compared to the conventional photonic crystal- or lattice-based approaches. For example, the energy of the flatband Bloch waves is completely independent of their wavevector $\boldsymbol{k}$, resulting in macroscopic degeneracy. While all flatbands necessarily have the same trivial dispersion relation $\omega(\boldsymbol{k})$ = constant, interestingly they do not all behave qualitatively the same. In fact, for practical reasons, flatbands can be grouped into three distinct types:

1. "Symmetry-protected" flatbands, corresponding to localized "dark" states decoupled from propagating channels
2. "Accidental" flatbands formed by fine-tuning of system parameters

3. "Topologically protected" flatbands, which are robust under perturbations to coupling parameters

These different types of flatbands are distinguished by considering the properties of their eigenstates. In conventional periodic media, the energy eigenstates are the delocalized Bloch waves. Since in a flatband, all the Bloch waves are degenerate, any superposition of them can also form a valid eigenstate, and in particular, one can construct *localized* eigenstates; they are localized by interference, even in the absence of any disorder or confining potential.

A natural question arises: what are the smallest possible eigenstates that can be constructed in a given flatband? The obvious choice would be the band's Wannier functions, which are exponentially localized superpositions of all the Bloch waves. In fact, in flatbands, even stronger localization known as compact localization is possible, with the eigenstate amplitudes strictly vanishing except at a finite number of unit cells of the structure. The profiles of these compact localized states, e.g., the number of unit cells U excited by the smallest such state and whether they form a linearly independent set, can help characterize a given flatband.[19,20]

The above classifications, while following practical reasons, are missing some mathematical rigor. For specialists, symmetry-protected flatbands can be replaced by $U = 1$ flatbands with compact localized states occupying precisely one elementary unit cell of the lattice, thus forming an orthogonal set.[20] Accidental flatbands can be replaced by $U \geq 2$ flatbands with compact localized states extending beyond one unit cell, thus forming a non-orthogonal set, and sometimes even a linearly dependent set; they include the line graph models pioneered by Mielke,[21] as well as Tasaki's "decorated" lattices with a fine-tuned next-nearest-neighbour coupling.[22] The "topologically protected" flatbands occur in systems with a bipartite symmetry such as the Lieb lattice,[5] which can be divided into two sub-systems of different sizes which have no direct coupling between elements within the same sub-system.

A. Simple examples of flatbands

We now present examples of the three types of flatbands, focusing on the abstract tight binding or coupled mode Hamiltonians and deferring discussion of photonic realizations to Sec. III.

An example of a symmetry-protected flatband is the quasi-1D cross-stitch ladder, shown in Fig. 1(a), which is symmetric under reflections about the ladder axis. This can be described by the tight binding equations,[20]

$$\delta\omega\psi_n^a = J_1(\psi_{n-1}^a + \psi_{n-1}^b + \psi_{n+1}^a + \psi_{n+1}^b) + J_2\psi_n^b, \tag{1}$$

$$\delta\omega\psi_n^b = J_1(\psi_{n-1}^a + \psi_{n-1}^b + \psi_{n+1}^a + \psi_{n+1}^b) + J_2\psi_n^a, \tag{2}$$

where $\delta\omega$ is a detuning with respect to some reference frequency, J_1 and J_2 are inter- and intra-cell coupling strengths, and ψ_n^a and ψ_n^b are amplitudes on the two legs in the nth unit cell. Transforming to basis states $\psi_n^+ = (\psi_n^a + \psi_n^b)/\sqrt{2}$ and $\psi_n^- = (\psi_n^a - \psi_n^b)/\sqrt{2}$ which are symmetric and antisymmetric under reflection, the eigenvalue problem simplifies to

$$\delta\omega\psi_n^+ = J_2\psi_n^+ + 2J_1(\psi_{n-1}^+ + \psi_{n+1}^+), \tag{3}$$

$$\delta\omega\psi_n^- = -J_2\psi_n^-, \tag{4}$$

and one can read off the Bloch wave energy eigenvalues as $\delta\omega = J_2 + 4J_1\cos(ka)$ and $\delta\omega = -J_2$, where k is the transverse wavevector and a is the array period. The latter forms the flatband with

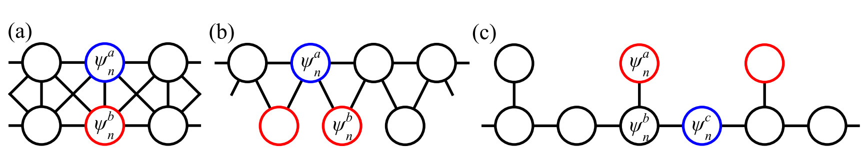

FIG. 1. Examples of quasi-1D flatband lattices. (a) Cross-stitch lattice with an inversion symmetry-protected flatband. (b) Sawtooth lattice with an accidental flatband. (c) Stub lattice with a flatband protected by the topology of the inter-site hoppings. Circles indicate sites with local field amplitudes ψ_n^j, and lines indicate nonzero inter-site hopping terms. Sites excited by the flatband's compact localized states are highlighted in red and blue, indicating phases of 0 and π, respectively.

energy independent of k. Tuning the intra-cell hopping J_2 preserves the flatband while shifting its energy with respect to the dispersive band. Perturbations that break the reflection symmetry of the ladder will couple the two bands and induce dispersion in the flatband.

The sawtooth ladder shown in Fig. 1(b) hosts an accidental flatband. It is described by the tight binding model,

$$\delta\omega\psi_n^a = J(\psi_{n-1}^a + \psi_{n+1}^a + \psi_{n-1}^b + \psi_n^b), \tag{5}$$

$$\delta\omega\psi_n^b = M\psi_n^b + J(\psi_n^a + \psi_{n+1}^a), \tag{6}$$

where M is a detuning of the b site energies. Both energy eigenvalues,

$$\delta\omega = \frac{M}{2} + J\cos(ka) \pm \sqrt{2J^2 + (M/2)^2 + J\cos(ka)(J\cos(ka) + 2J - M)}, \tag{7}$$

typically depend on k. However, at the critical detuning $M = -J$, the term inside the square root of Eq. (7) becomes the perfect square $(J\cos(ka) + M/2 + 2J)^2$, resulting in a flatband with energy $\delta\omega = -2J$. Actually, this flatband is not an isolated example but forms a parametric family under variation of the relative strengths of the inter- and intra-leg hopping terms. With fine-tuning of parameters, one can obtain similar flat or nearly flatbands in multi-mode Hamiltonians.[23,24]

The third class of "topologically protected" flatbands occurs in bipartite systems, which can be divided into two classes of sublattices, called "majority" and "minority." Sites belonging to each are not directly coupled to one another; hopping between majority sites can only occur via a minority site, and vice versa. This imbues a chiral symmetry on the Hamiltonian: for each mode with energy $\delta\omega$, there is a mode with energy $-\delta\omega$. Consequently, the excess modes of the majority lattice must lie at $\delta\omega = 0$, forming a flatband.[25] Moreover, the chiral symmetry and flatband persist even under disorder to the coupling strengths $J \to J_n$; the flatband is protected by the connectivity of the sublattices, not the precise values of the coupling strengths. An example of such a bipartite lattice is the stub lattice, shown in Fig. 1(c) and described by the tight binding model,

$$\delta\omega\psi_n^a = J_1\psi_n^b, \tag{8}$$

$$\delta\omega\psi_n^b = J_1\psi_n^a + J_2\psi_{n-1}^c + J_3\psi_n^c, \tag{9}$$

$$\delta\omega\psi_n^c = J_2\psi_{n+1}^b + J_3\psi_n^b \tag{10}$$

with the energy eigenvalues $\delta\omega = 0, \pm\sqrt{J_1^2 + J_2^2 + J_3^2 + 2J_2J_3\cos(ka)}$ including the promised flatband which is independent of the coupling strengths $J_{1,2,3}$.

Figure 1 also shows the compact localized states support by each type of flatband. For the symmetry-protected flatband, the decoupled antisymmetric states are independent of k, such that the Wannier functions and compact localized states are identical, residing in a single unit cell; the flatband can therefore be considered as a simple array of independent bound states in the continuum.[26] In accidental and topologically protected flatbands, the Bloch wave profiles depend on k such that the Wannier functions are only exponentially localized, and the compact localized states excite a minimum of two unit cells. This means that local perturbations, e.g., induced by nonlinearities, will typically couple different compact localized states, enabling novel interaction-induced wave transport regimes and, in quantum systems, interesting many-body quantum phases. On the other hand, the symmetry-protected flatband transport can only be mediated by first coupling into dispersive waves, resulting in a simpler behavior. In other examples, particularly in higher dimensional flatband lattices, the minimal compact localized states can span several unit cells.

III. EXPERIMENTS WITH PHOTONIC FLATBANDS

We now discuss how the abstract models presented above have been realized in various experiments and some potential applications of flatbands and their peculiar localized eigenstates.

A. Waveguide arrays

In optical waveguide arrays, the propagation direction is fixed, with the distance along the propagation axis playing the role of "time," and the evolution of the field in the transverse plane is of primary interest. For the commonly used case of weak refractive index modulations, light propagation in waveguide arrays is described by the paraxial wave equation for the field envelope $\psi(x, y, z)$ slowly varying along the waveguide axis z,

$$i\partial_z \psi = \left[-\frac{1}{2k_0}\nabla_\perp^2 - \frac{k_0}{n_0}\Delta n \right] \psi, \tag{11}$$

where $k_0 = 2\pi n_0/\lambda$ is the wavenumber, n_0 is the ambient refractive index, $\nabla_\perp = \partial_x^2 + \partial_y^2$ is the transverse Laplacian, and Δn describes the index profile of the waveguide array. Popular methods to create a desired 2D [$\Delta n = \Delta n(x, y)$] or 3D [$\Delta n = \Delta n(x, y, z)$] index modulation are optical induction[27] and direct laser writing.[16] The latter has the advantage of allowing precise fine-tuning of the lattice geometry and inter-waveguide coupling strengths, allowing fabrication of many quasi-1D and 2D flatband lattices.

We stress that in this setting, the eigenvalue problem Eq. (11) is for the propagation constant k_z, i.e., one looks for propagation-invariant eigenstates $\psi(z) = \psi_0 e^{-ik_z z}$ at a fixed frequency $\omega = 2\pi c/\lambda$. Here k_z plays the role of "energy" instead of the mode frequency; therefore, flatbands in paraxial waveguide arrays do not lead to genuine slow light but rather suppression of transverse diffraction.

When neighbouring waveguides are weakly coupled, a tight binding approximation can be applied and continuous Eq. (11) reduces to discrete tight binding equations similar to those presented in Sec. II, with the waveguide separation controlling the effective coupling strengths J_n. The waveguide depth (detuning) can also be tuned independently, providing great flexibility in the realization of 1D or 2D tight binding models with short range couplings and allowing demonstration of various quantum-optical analogies.[28]

In fact, the first demonstrations of flat dispersion in waveguide arrays were based on an analogy with the dynamic localization of electrons under high-frequency electric fields,[29] which is mimicked by a uniform periodic z modulation of the waveguide positions, e.g., $\Delta n = \Delta n(x - x_0(z))$, where $x_0(z)$ is the modulation profile.[30] Under this high-frequency driving, the effective coupling strength J is renormalized to $J_{\text{eff}} = J \int_0^L dz \exp[-i\gamma \partial_z x_0(z)]$, where L is the modulation period and γ is a constant dependent on the lattice period and operating wavelength. By careful tuning of the modulation parameters, one can achieve $J_{\text{eff}} = 0$, resulting in a completely flat transverse dispersion relation and restoration of the optical field to its initial profile at integer multiples of the modulation period. Optimizing the modulation profile $x_0(z)$, broadband and two-dimensional dynamic localization have also been demonstrated.[31–33]

The first demonstrations of a static flatband lattice Hamiltonian in waveguide arrays were based on the laser-written 2D Lieb lattice shown in Fig. 2.[34–36] Its flat or dispersive band states can be

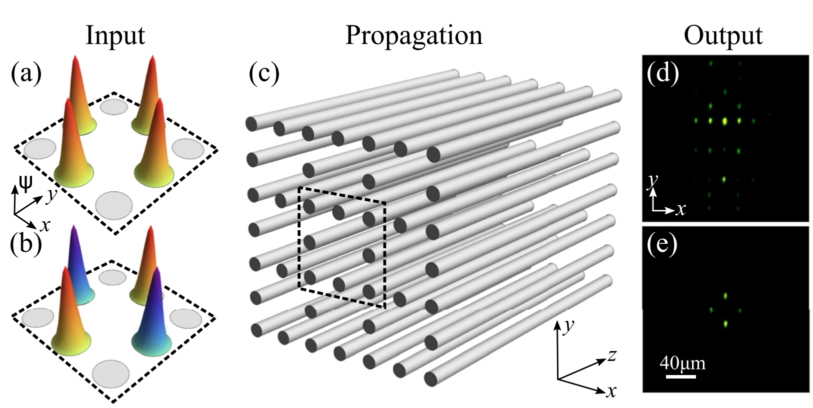

FIG. 2. Laser-written photonic Lieb lattice hosting a flatband. [(a) and (b)] Transverse profiles of the multi-waveguide input states. In (a), the phase is uniform, resulting in excitation of dispersive bands. In (b), the phase is staggered, alternating between 0 (orange) and π (blue), exciting the flatband. (c) Dynamics are probed by propagation of the input states along the array (z) axis. [(d) and (e)] Experimentally measured output intensity profiles adapted from R. A. Vicencio *et al.*, Phys. Rev. Lett. **114**, 245503 (2015). Copyright 2015 American Physical Society. (d) Strong diffraction is seen for the dispersive band input (a). (e) The compact localized state (b) remains propagation invariant.

selectively excited by tuning the phase of the input beam $\psi(x, y, 0)$ that excites waveguides of the majority sublattice formed by the "edge" sites which lie between the "corner" sites. After propagation through the array, a dispersive band wavepacket will experience dephasing between its different transverse wavevector components $k_z = k_z(k_x, k_y)$, resulting in diffraction and the delocalized output state such as the one shown in Fig. 2(d). On the other hand, within the nearest neighbour tight binding approximation, the flatband states remain propagation invariant and the input state is preserved, see Fig. 2(e). The Lieb lattice flatband is protected by the topology of the inter-site couplings and not their precise strengths, and the robustness of the flatband to coupling disorder (introduced by misalignment of the waveguide positions) was demonstrated by Mukherjee *et al.*[36]

The original publications observing compact localized states in the Lieb lattice attracted widespread interest, and demonstrations of compact localized states in several other 2D lattices,[37,38] quasi-1D lattices,[39–42] and higher waveguide modes[43] quickly followed suit. Among these, an especially active area has been the exploration of flatbands in periodically driven Floquet lattices in 1D[44] and 2D.[45,46] Floquet lattices generalize the dynamic localization approach discussed above to have independent control over the individual waveguide positions and depths, allowing realization of near-arbitrary time-dependent Hamiltonians described by Floquet band structures. For example, carefully tuned driving can produce an effective magnetic flux and induce a completely flat spectrum, despite the effective inter-waveguide couplings remaining nonzero. This peculiar magnetic field-induced localization, known as the Aharonov-Bohm caging,[47] was very recently observed in two experiments.[48,49]

Two proposed applications of these waveguide arrays are quantum simulation of strongly correlated states of matter and prototyping novel designs for optical fibres and photonic bound states in the continuum, where the non-diffracting nature of the compact localized states may be useful for suppressing unwanted cross talk between different channels within a photonic crystal fibre.[35]

Presently, optical waveguide arrays are effectively limited to probing linear dynamics. In principle, the nonlinear propagation regime in laser-written structures is accessible by probing the system with a pulsed laser, and discrete solitons were already observed by Szameit *et al.* in simple square lattices more than 10 years ago.[50,51] However, in practice the probe power is limited by the need to avoid the inducing material damage. Szameit *et al.* circumvented this by fabricating arrays with a very weak coupling, which lowers the power required but limits the nonlinear dynamics to short effective propagation lengths, making it hard to distinguish between the flat and dispersive bands.

While optically induced lattices can display a much stronger nonlinear response observable with continuous wave beams,[27] a weak inter-waveguide coupling is also required to minimize the unwanted next-nearest neighbour hopping and ensure faithful induction of the flatbands. Hence, any nonlinear dynamics are similarly limited to rather short propagation distances.

Therefore, simulating novel strongly interacting phases of matter with the existing flatband waveguide arrays remains challenging, and novel approaches such as state recycling to increase the effective propagation length,[52] inclusion of nonlinear dopants,[53] or laser-writing in highly nonlinear materials such as chalcogenides[54] will need to be explored to make further progress in this direction.

On the other hand, in the linear regime, there are still several directions that deserve further attention, including the Landau-Zener tunneling between flat and dispersive bands,[55–57] disorder-induced dephasing of flatband states,[58] propagation of entangled states of light,[59] and non-Hermitian or parity-time symmetric flatband lattices,[60–67] which can be implemented passively by rapidly bending the waveguides to induce loss.[68] Research along these lines, while not expected to directly lead to technological applications, may provide novel insights into other systems such as frustration in condensed matter,[69] optically driven electronic flatbands, or novel designs for photonic crystal fibres.

B. Exciton-Polaritons

Exciton-polaritons are quasiparticles arising from a strong light-matter coupling in semiconductor microcavities. The low effective mass provided by their photonic component combined with an exciton-mediated nonlinear response is ideal for demonstrating effects such as the Bose-Einstein condensation and low-power optical switching. There are a variety of methods to induce structured potentials for the exciton-polaritons, allowing the creation of periodic lattices and flatbands.[70]

Compared to waveguide arrays, a distinguishing feature of exciton-polaritons is their finite lifetime, caused by the exciton recombination and leakage of photons from the cavity, forming a driven-dissipative system. In the presence of incoherent optical pumping, the time evolution of the polariton condensate wavefunction Ψ is governed by the nonlinear Schrödinger equation,[70]

$$i\hbar\partial_t \Psi = \left[-\frac{\hbar^2}{2m}\nabla_\perp^2 + g_c|\Psi|^2 + g_R n_R(\boldsymbol{r},t) + U(\boldsymbol{r}) + i\frac{\hbar}{2}(R n_R(\boldsymbol{r},t) - \gamma)\right]\Psi, \tag{12}$$

$$\partial_t n_R = -(\gamma_R + R|\Psi|^2) n_R(\boldsymbol{r},t) + P(\boldsymbol{r}), \tag{13}$$

where m is the polariton effective mass, g_c and g_R are polariton-polariton and polariton-reservoir interaction strengths, n_R is the reservoir density, $U(\boldsymbol{r})$ is the lattice potential, R is the stimulated scattering rate, γ and γ_R are polariton and reservoir exciton loss rates, and $P(\boldsymbol{r})$ is the spatial profile of the optical pump.

At low pump powers, below the condensation threshold, the polariton density term $|\Psi|^2$ is negligible and the linear band structure of Eq. (12) can be measured from the photoluminescence spectrum of the microcavity. The first successful application of this approach to flatbands was reported in 2014 by Jacqmin *et al.*, using a honeycomb lattice formed by etched micropillars.[71] They found that the higher P-orbital bands of the micropillars could be described by a kagome lattice-like tight binding model and observed its flatband and the real-space Bloch wave profiles and in a later publication observed corresponding edge states.[72] We note that an earlier kagome lattice structure fabricated using metallic film deposition by Masumoto *et al.* was not sufficiently deep to allow resolution of the flatband.[73]

At higher pump powers, condensation occurs and interesting interaction effects induced by the nonlinear terms of Eq. (12) become observable. However, under off-resonant pumping, one cannot directly control into which state the system condenses; in the above examples, condensation into a dispersive band was observed rather than the flatband. Baboux *et al.* demonstrated a solution to this problem in 2016,[74] using a quasi-1D stub lattice formed by etched micropillars shown in Fig. 3(a). In the stub lattice, the flatband is sandwiched between two dispersive bands, as shown in the measured photoluminescence spectrum in Fig. 3(b). By changing the spatial structure of the optical pump, Baboux *et al.* could control the relative gain of the dispersive and flatband states. In particular, with a uniform pump, condensation into the upper dispersive band was observed [Fig. 3(c)], while a pump localized to only the sublattice of "A" pillars produced condensation into the flatband [Fig. 3(d)]. Notably, one can see in Figs. 3(c) and 3(d) that the dispersive band condensate is delocalised in real space, remaining coherent over many lattice sites, while the flatband fragments into multiple

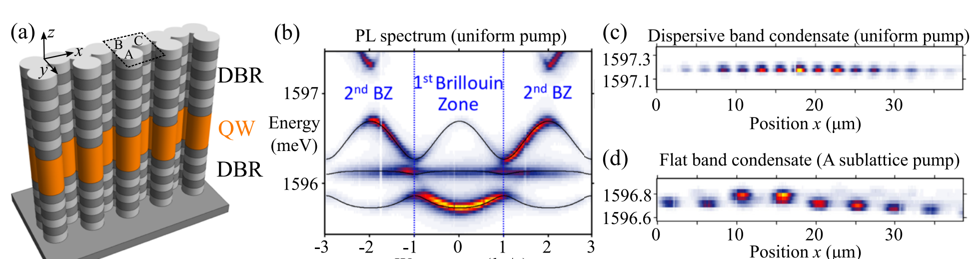

FIG. 3. Flatbands in coupled micropillar cavities, adapted from F. Baboux *et al.*, Phys. Rev. Lett. **116**, 066402 (2016). Copyright 2016 American Physical Society. (a) Coupled cavities formed by quantum wells (QWs) embedded between the distributed Bragg reflectors (DBRs). (b) Photoluminescence (PL) spectrum of a uniformly pumped array, revealing a flatband sandwiched between two dispersive bands. [(c) and (d)] Position-resolved emission spectrum of the condensates formed at high pump powers. (c) In a uniformly pumped array, condensation occurs into a single delocalized dispersive band state, maintaining phase coherence across many periods of the lattice. (d) When the pump is shaped to only excite the A sublattice, condensation into compact localized flatband states is favoured. Disorder lifts the degeneracy between different compact localized states, forming many mutually incoherent condensates with energies dependent on the local disorder potential.

incoherent compact localized states, with their energy degeneracy lifted by the intrinsic disorder of the lattice.

Two groups subsequently observed condensation into the flatbands of a 2D Lieb lattice, similarly observing fragmentation of the condensate by disorder.[75,76] In 2D lattices, the inter-site coupling strength becomes polarization-dependent, producing an effective spin-orbit coupling.[77] Consequently, the flatband condensates exhibit nontrivial polarization textures in their emission spectra. In the future, this combination of spin-orbit coupling, interactions, and frustrated lattice geometries is anticipated to allow the simulation of novel strongly correlated magnetic phases. For example, classical XY and Ising Hamiltonians have now been simulated on small lattices and plaquettes,[78,79] and the main challenge is to scale up to extended flatband lattices.

Another interesting avenue for future experiments is the behavior of the above flatbands under resonant optical pumping.[80–82] While transport is completely suppressed in a non-interacting flatband, interactions can induce transport resulting in long range correlations[80] and multiphoton states at the nominally dark sites neighbouring the compact localized states[81,82] under a pump resonant with the flatband.

In all cases, such strongly correlated states must compete with disorder, which favours the formation of incoherent fragments instead of a long range order. In principle, structured optical pumping can be used to compensate for the disorder via the polariton-reservoir interaction term in Eq. (12), although this may be difficult in practice. At the same time, the interaction strength should exceed the polariton lifetime, which requires high quality factor microresonators. Another challenge here is that methods required to create deep polariton potentials, such as etching, inevitably introduce additional losses.[70] Nevertheless, polaritons remain the most promising avenue for exploring strong interaction effects in photonic flatbands.

Finally, the interplay between non-Hermiticity and flat dispersion has scarcely been explored in this setting, with the exception of Ref. 74. In addition to the references mentioned above,[60–67] the pump-induced gain in flatband lattices can be used to realize novel lasing regimes.[83,84]

C. Metamaterials

While waveguide arrays and exciton-polariton condensates may be the better-known examples of photonic flatbands, they have also been realized with two classes of 2D metamaterials: spoof plasmons, and all-dielectric zero-index metamaterials. What distinguishes flatband states from conventional "dark" modes of metamaterials are that the latter can be localized to a single element (i.e., decoupled by symmetry, analogous to the $U = 1$ cross-stitch lattice), while localization in nontrivial flatbands of class $U > 1$ is a collective effect; the compact localized states cannot be reduced to a single unit cell. Consequently, the flatband Bloch modes acquire a strong $\boldsymbol{k}$-dependence.[85]

The group of Kitano has performed experiments with terahertz spoof plasmons in kagome and Lieb lattices fabricated from etched steel sheets.[86–88] They considered 2D flatband lattices forming metasurfaces which are probed at oblique incidence, as shown in Fig. 4(a). Inside (outside) the light line, the flatband hosts an angle-independent transmission minimum (reflection maximum), with the quality factor of the flatband states being sensitive to the input angle. In particular, at normal incidence, the flatband modes become "dark" states completely decoupled from the incident beam. The corresponding experimentally measured transmission and reflection spectra for the Lieb lattice are shown in Fig. 4(b), where the next-nearest-neighbour coupling unflattens the band at large incidence angles.

Concurrently, in 2011, Huang *et al.* introduced a class of zero-refractive-index all-dielectric metamaterials based on fine-tuning to induce degeneracy between dipolar and monopolar modes in a square lattice of dielectric rods, producing an intersection between flat and conical bands which resembles the dispersion of the Lieb lattice.[89] This original microwave experiment has attracted significant interest and has now been scaled up to optical frequencies.[90,91] In these experiments, propagation is confined to the plane of the 2D lattice and the zero effective refractive index conical bands mediate near-unity transmission for normally incident beams, even in the presence of obstacles or corners. Excitation at oblique incidence, however, results in an unwanted reduction in transmission and loss of zero-index behavior due to the excitation of flatband modes, which is enabled by the band not being perfectly flat.[92]

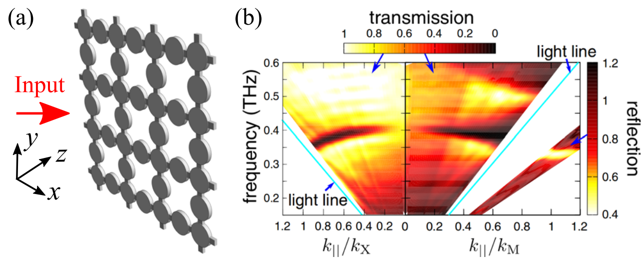

FIG. 4. Lieb lattice for spoof surface plasmons. (a) Schematic of the scattering experiments. The input beam is incident at a variable angle with respect to the plane of the lattice. (b) Measured transmission and reflection as a function of frequency and input angle with respect to the X ($k_{\|}/k_X$) and M ($k_{\|}/k_M$) points of the Brillouin zone, reproduced with permission from S. Kajiwara *et al.*, Phys. Rev. B **93**, 075126 (2016). Copyright 2016 American Physical Society. The transmission minimum (inside the light line) and maximum (outside the light line) correspond to excitation of the flatband modes.

Presently, these metamaterial studies are relatively disconnected from the flatband literature discussed in Secs. II, III A and III B, and it is interesting (for us, at least) to offer some poorly informed speculation as to how those results may be applied here. For example, the role of the flatband class, e.g., its effect on the quality factor of the dark modes, has not been systematically studied nor have the exact differences between compact localized flatband states and conventional dark modes.

Closely related to the above experiments is the concept of "resonant guided wave networks," which were introduced by Feigenbaum and Atwater in 2010[93,94] and demonstrated in small plasmonic networks in 2014.[95] Wave localization in these networks resembles localization in "topologically protected" flatbands such as the Lieb, dependent only on the local connectivity between different waveguides. It would be interesting to explore this analogy further.

Introducing localized defects to a zero index metamaterial will couple the flatband states to the dispersive bands and may induce strong Fano resonances. Another interesting question is how the behavior of such defects compares to the recently demonstrated "doping" of epsilon-near-zero media:[96] both can be considered a localized perturbation to an otherwise zero index effective medium.

Finally, in 2012 Nakata *et al.*[97] proposed flatbands in metamaterials modelled as inductor-capacitor circuits, such as transmission line metamaterials. In contrast to the standard tight binding approach, tightly bound eigenmodes at individual lattice sites are not required, allowing realization of flatband models with longer range couplings such as the seminal "decorated" lattice models first studied by Tasaki in 1992.[22] These models are presently inaccessible with waveguide arrays and exciton-polaritons, which are limited to tight binding models with short-ranged evanescent couplings. To our knowledge, flatbands based on circuit or transmission line metamaterials have not yet been observed.

IV. FUTURE DIRECTIONS

We now turn to platforms where flatbands have not been extensively explored yet, but we believe they deserve more attention in the future.

A. Coupled resonator lattices

Coupled resonator lattices can be implemented using various methods, such as arrays of defects in photonic crystals[98,99] and coupled microring resonators.[100,101] Propagation is governed by tight binding models similar to those appearing in Sec. II in the limit of weak inter-resonator coupling, and in general by transfer matrices.[102] However, we are not aware of any experimental demonstrations of flatbands in this platform to date; most studies have focused on the characterization and optimization of the simple Bravais lattices.

A notable exception is the 2013 paper by Hafezi *et al.*,[103] which considered a bipartite lattice of ring resonators shown in Fig. 5(a) consisting of resonant site rings coupled by off-resonant link rings. While the focus of Ref. 103 was to demonstrate topological edge states, the lattice used bears a striking resemblance to the 2D Lieb lattice of Fig. 2 and therefore shows promise for implementing flatbands. In fact, a similar design proposed by Zhu *et al.*[104] early this year based on a bipartite

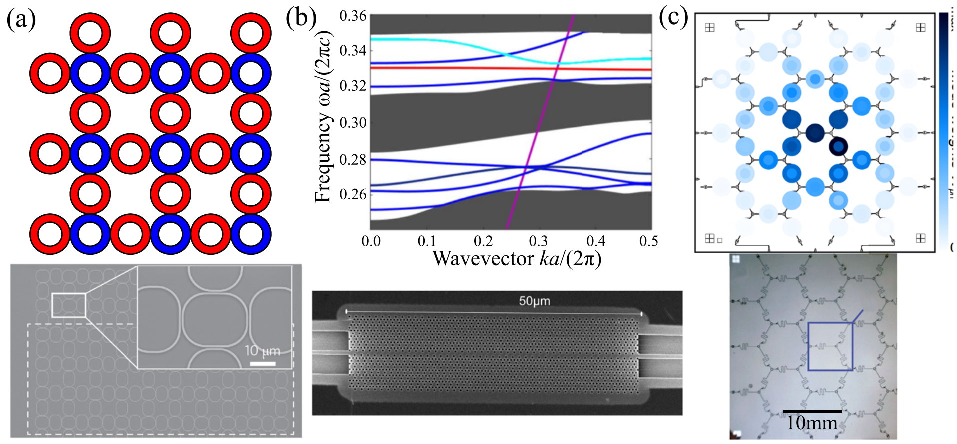

FIG. 5. Novel platforms for exploring photonic flatbands. (a) Coupled resonator lattices. Top: Lieb-like lattice formed by detuned "site" (blue) and "link" (red) rings. Bottom: scanning electron microscope image of the lattice, adapted from M. Hafezi *et al.*, Nat. Photonics **7**, 1001 (2013). Copyright 2013 Macmillan Publishers Limited. (b) Photonic crystals. Top: Simulated band structure of 1D photonic crystal waveguide formed by removing a row of holes from kagome photonic crystal. $a = 500$ nm is the hole spacing. Gray shaded regions are bulk bands, and curves indicate defect modes. Flatband modes in the guided region outside the light line (purple) are highlighted in red and cyan. Bottom: Fabricated photonic crystal membrane. Adapted from S. A. Schulz *et al.*, Opt. Lett. **42**, 3243 (2017). Copyright 2017 Optical Society of America. (c) Microwave circuit QED. Top: Space-resolved imaging of a dispersive band eigenstate of a kagome lattice of 49 coupled microwave resonators. Bottom: Picture of the device. Adapted from D. L. Underwood *et al.*, Phys. Rev. X **6**, 021044 (2016). Published by the American Physical Society under the Creative Commons Attribution 3.0 License.

honeycomb lattice supports a nearly flatband co-existing with topological edge states. We believe this is an exciting development.

In another recent theoretical study by one of us,[105] we found that off-resonant link rings can implement a strong next-nearest neighbour coupling, which would allow models such as the cross-stitch or sawtooth of Fig. 1(a) to be realized. This approach could be further generalized to longer range couplings, to implement Tasaki's decorated lattice models.[22]

A pressing issue important for potential applications of resonator flatbands is to understand how the reduced group velocity in flatband resonator lattices compares to the traditional approaches for achieving structural slow light based on weakly coupled resonators. The interesting potential advantage offered by flatbands is that the group velocity v_G can be tuned independently of the bare coupling strength J, allowing more flexibility to optimize parameters to minimize unwanted effects such as two-photon absorption or disorder-induced localization and dephasing. Moreover, the high light intensities accessible in coupled resonator lattices provide a promising setting for exploring interaction and quantum effects in flatbands.

Moving beyond Hermitian limits, coupled optical resonators provide a novel setting for re-examining lasing in frustrated lattice geometries. The original experiment by Nixon *et al.*[83] in 2013 required intricate alignment of a single cavity, patterning holes in the cavity mirrors to induce a flatband lattice. Using integrated resonator arrays would allow frustrated lasing to be studied in a more stable platform. We note that lasing in topological microring resonator arrays was demonstrated last year,[106–109] but the behavior of flatband arrays remains completely unexplored. Will the resulting lasing mode display long range correlations similar to those observed in Ref. 83, or will disorder inevitably result in short range-correlated, multimode lasing? Driven-dissipative flatbands in integrated photonics are a largely unexplored area that deserves future theoretical and experimental studies.

B. Photonic crystals

The first flatband-inspired photonic crystal slab design by Takeda *et al.* in 2004 consisted of a 2D kagome lattice of high-index dielectric rods well approximated by a tight binding model.[110] Their selling point was the isotropic response of their design at the flatband frequency; at this frequency, any wavevector can be resonant with a Bloch mode. However, this proposal received little attention compared to air hole structures, which were much easier to fabricate. For several years, designers of flatbands for slow light applications mainly focused on the numerical or intuitive optimization of 1D photonic crystal waveguides.[111,112]

The recent progress in achieving flatbands with waveguide arrays, exciton-polaritons, and metamaterials has led to a revival in interest of flatband lattice-inspired designs for photonic crystals.[24,113–115] For example, in 2015 Xu *et al.*[113] applied an optimization procedure to the original zero-index metamaterial of Ref. 89 to increase the band flatness over the entire 2D Brillouin zone. Their optimized design consisted of a bipartite lattice of "corner" and "edge" dielectric rods of different radii, resembling a Lieb lattice. Similarly, Nguyen *et al.*[24] used the coupled mode theory to analytically optimize symmetry-breaking to induce an accidental flatband. They experimentally demonstrated a 1D photonic crystal for near-infrared wavelengths with a flatband formed by fine-tuned interference between even and odd modes.

Quasi-1D flatband lattices may also inspire novel photonic crystal waveguide designs. A 2017 study by Schulz *et al.*[114] considered the inverse structure to that of Takeda *et al.*: a kagome lattice of air holes in a slab. The kagome lattice can be constructed by removing rods from a regular triangular lattice, and its band structure is qualitatively understood in terms of a tight binding model for the missing rods. This approach is therefore intermediate between regular photonic crystals and coupled resonator lattices formed by arrays of photonic crystal defects.[98,99] They found that the enlarged unit cell not only compresses the bulk bands, reducing their group velocity, but also provides more degrees of freedom for the optimization of photonic crystal defect waveguides. Figure 5(b) illustrates their fabricated sawtooth-like defect waveguide formed by removing lines of holes from the structure, exhibiting two guided slow light modes. However, it is still not clear whether the resulting waveguides can be related to quasi-1D flatbands or the mechanism is simply the denser band structure created by the enlarged unit cell. This is a question which deserves further investigation.

Any flatband-based approaches for designing slow light photonic crystal waveguides will of course need to be judged against traditional approaches and optimization methods.[111,112] Compared to the simplest triangular lattice, typical flatband lattice geometries will have a larger structural period, resulting in a red shift in the photonic bands for a fixed index contrast and folding of the Brillouin zone, bringing modes into resonance with the light cone. Likely the improved group velocity offered in flatband lattice-inspired designs will come at the cost of reduced bandwidth, with their main advantage likely being reduced group velocity dispersion.

Finally, we note that there are so far no systematic studies of flatbands in three-dimensional photonic crystals. While challenging to fabricate for optical wavelengths, even demonstrations at microwave frequencies would open a completely new direction by allowing exploration of effects such as the disorder-induced inverse Anderson transitions[116] which are inaccessible in lower-dimensional systems and all the other material platforms we have discussed.

C. Microwave circuit QED

The final platform we would like to briefly mention is circuit quantum electrodynamics using superconducting microwave circuits.[117] This setting offers the unique opportunity to explore the strongly interacting quantum regime of flatbands in photonics, with potential applications including quantum simulation and computing. While there have been a few implementations of frustrated kagome lattices since 2016,[118,119] direct imaging of the flatband and its many-body dynamics remains elusive. A challenge is that experiments must be conducted at cryogenic temperatures and they rely on indirect measurements of scattering spectra, which makes it especially hard to measure bulk flatband states and distinguish them from edge resonances. Nevertheless, there has been significant experimental progress in realizing and probing high quality, low disorder lattices approaching hundreds of sites since 2012.[120–122]

For example, in 2016, Underwood *et al.*[118] demonstrated a method to image the interior of 2D circuit QED lattices. They used a movable probe to induce a weak defect at a single lattice site. They showed that the influence of this defect on the lattice's transmission spectrum can be used to indirectly measure the number of photons at the probed resonator and therefore produce an image of the intensity throughout the lattice. Figure 5(c) illustrates the application of this method to reconstruct the profile of a dispersive band eigenstate in a 49 site kagome lattice. One limitation of this approach that will need to be solved in the future is that the imaging is based on measuring the transmission spectrum. Since the flatband states exhibit poor transmission, they will be limited to a poor signal-to-noise ratio; Underwood *et al.* did not present any measurements of flatband states using this technique.

In the meantime, there have been several theoretical studies studying the phase transitions, interactions, the role of dissipation, and quantum correlations in circuit QED flatbands,[80–82,123–126] largely focusing on small quasi-1D systems which remain numerically tractable in the quantum many-body regime. As imaging methods improve, we hope some of these effects will become observable in the near future.

One novel advantage offered by circuit QED is that the inter-site coupling strength becomes independent of the physical distance between the lattice sites. This enables the realization of exotic tight binding models, such as the kagome lattice on curved space, where it was shown that curvature can gap the flatband.[119] Using a similar approach, it should also be possible to experimentally observe flatbands in aperiodic networks such as the Penrose lattice.[7]

V. CONCLUDING REMARKS

In all the photonic systems we have discussed, flatbands represent an ideal limit. Disorder, systematic errors in fabrication, or corrections beyond the commonly used tight binding approximation will inevitably induce perturbations and introduce a cutoff to the flatband phenomena of interest, e.g., by inducing slight dispersion or the Anderson localization. Nevertheless, we strongly believe that this ideal is worth pursuing, not only as a means of characterizing fabrication methods and sources of imperfections but also because of their potential for demonstrating novel phases of light using flatbands and perhaps eventually device applications such as lasers, amplifiers, or quantum light sources. We therefore hope that the interesting open problems we have highlighted in this Perspective will convince others to go flat out with studying photonic flatbands.

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